Lecture 1: Causal Identification

POL-GA 1251
Quantitative Political Analysis II
Prof. Cyrus Samii
NYU Politics

January 29, 2020

Plan for the week

Mon:

- Discuss overview of class and syllabus.
- Explain what "causal identification" means.
- Introduce potential outcomes and causal graphs.

Wed:

- Randomized experiments.
- Explain *estimation* concepts (estimand, estimators, bias, consistency, efficiency).
- Explain *statistical inference* concepts (sampling distribution, randomization distribution, CLT, confidence intervals, p-value).

Where this class fits in

Model of quantitative research process:

- Theory motivates causal hypothesis or target of inference:
  - *H: manipulating X results in (...) effect on Y.*
- Hypothesis, statistical theory, and substantive theory motivate a research design:
  - Operationalize X and Y.
  - Define ways to get optimal variation in X and Y given constraints.
- Research design and statistical theory motivate analysis plan:
  - Optimal estimation strategy, given constraints.
  - Optimal testing strategy, given constraints.

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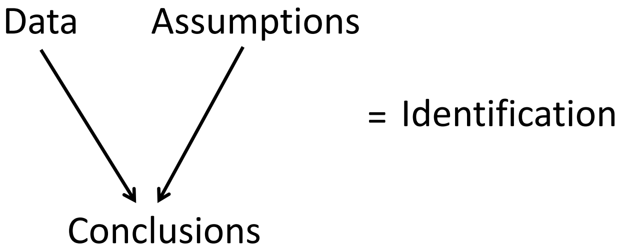

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In this class we focus on causal identification. This is a specific application of the general idea of "identification," distinct from some other applications:

What do we mean by “identification”?

Alternative application of “identification” (I):

Suppose someone says...

...they prefer Klobuchar over Sanders, and

...they prefer Warren over Klobuchar.

Does this information (data) identify the person’s preference ordering over these three candidates?

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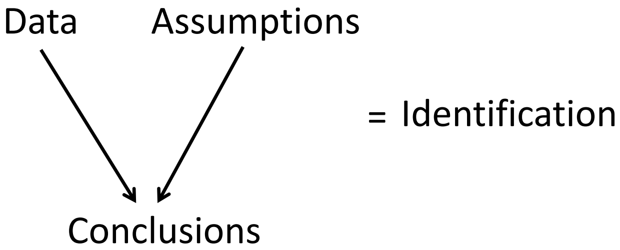

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Alternative application of "identification" (II):

Suppose none of the coefficients below are equal to zero but the error terms (last ones) are iid mean zero draws. Which system of simultaneous equations identifies its coefficients?

$$x_t = \alpha_1^a + \alpha_2^a y_t + \nu_t^a$$
$$y_t = \beta_1^a + \beta_2^a x_t + \varepsilon_t^a$$

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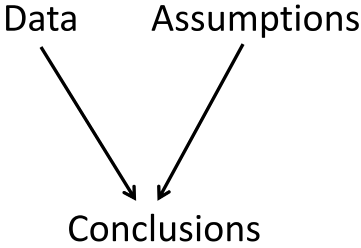

= Identification

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Angrist and Krueger (1999):

> *The combination of a clearly labeled source of identifying variation in a causal variable and the use of a particular econometric[/statistical] technique to exploit this information is what we call an identification strategy.*

Potential outcomes

The Road Not Taken

Two roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

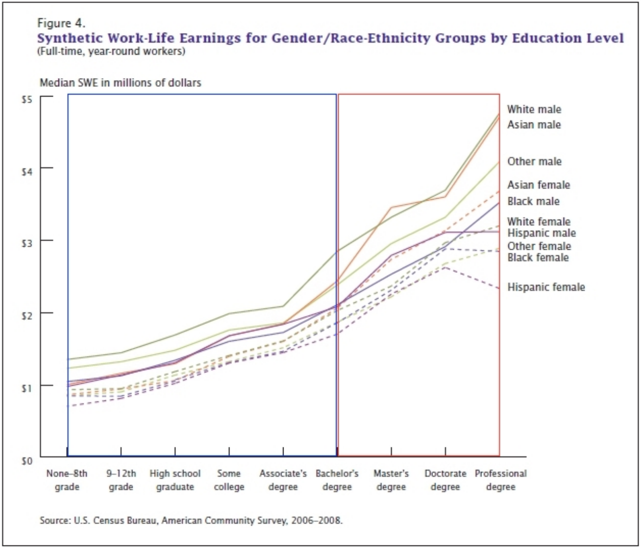
I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.

Modern frameworks for causal analysis:

- Potential outcomes (Neyman, 1923; Rubin, 1974, 1978).
- Causal graphs (Pearl, 2009).

Both rely on “counterfactual” logic.

Running Example: Effect of College on Earnings*



*SWE = expected total earnings over 25-64.

Potential outcomes

A causal effect can be defined as
a contrast between "potential outcomes."

Potential outcomes

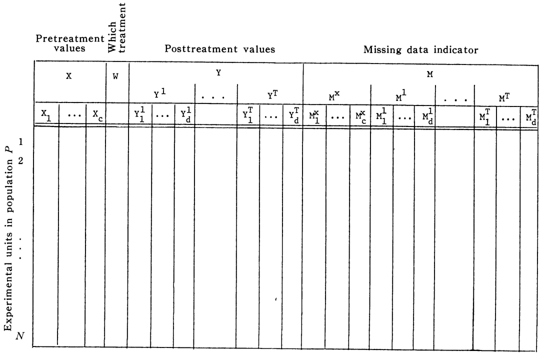


FIG. 1. All values in a study of T treatments.

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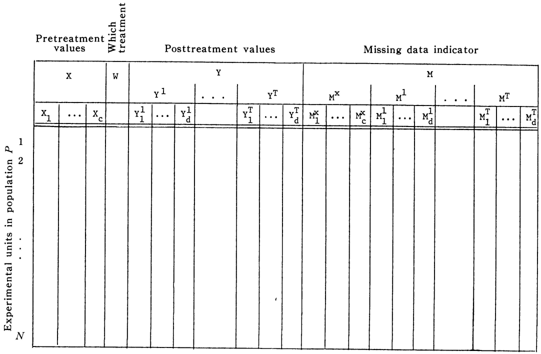

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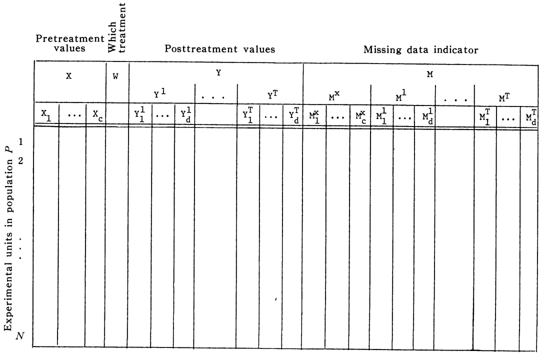

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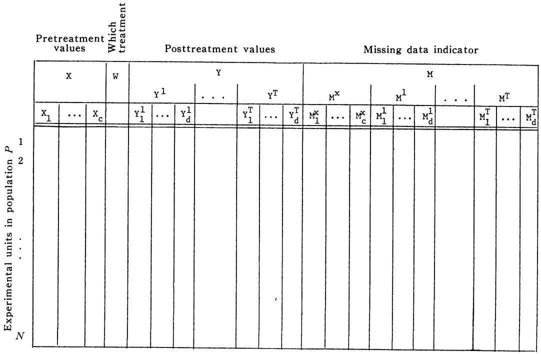

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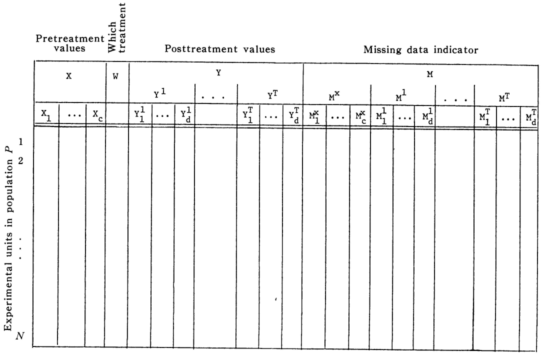

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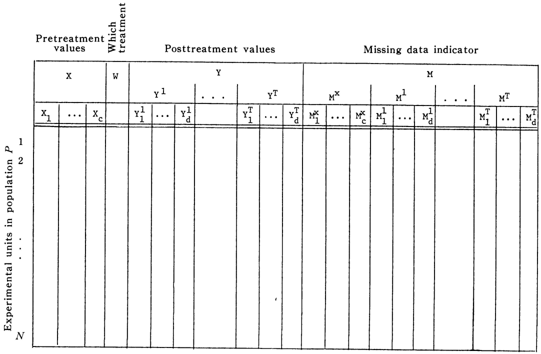

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- Missing data indicators, $M_{i,j}^{(k)}$.

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- *Population causal effects* for compare aggregates of unit level causal effects for members of $\mathscr{P}$.
- Effects are defined in an "agnostic" or "non-parametric" way.
- Potential outcomes and covariates are fixed, treatments and response indicators stochastic.
- Effects are defined by letting only treatments vary, holding units fixed.
- Thus, causal effects are clearly defined for units that can conceivably receive different treatment values.
- A test for the above is "manipulation" (Holland, 1986).

Potential outcomes, causal effects, and manipulability

Holland (1986) : "For causal inference, it is critical that each unit be potentially exposable to any one of the causes."

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Issues arise when trying to interpret things like race or gender. See VanderWeele and Robinson (2014) for a formal treatment of ways to interpret "race effects."

Potential outcomes and fundamental problem of causal inference

Recall, a unit level causal effect compares y_{wi} to $y_{\tilde{w}i}$ for $w \neq \tilde{w}$.

"Fundamental problem of causal inference" (Holland, 1986) : For each i potential outcomes for all w exist, but we only observe the potential outcome for the treatment value that i receives.

- "Scientific solution": Use theory to determine when units are interchangeable.
- "Statistical solution": Study features of conditional distributions, such as averages.

Causal identification under the potential outcomes model

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- Consider a random draw, i, from $\mathscr{P}$, countable but large.
- Each draw is characterized by
  - a covariate vector, X_i,
  - potential outcomes that under SUTVA are characterized as Y_{di} for all $d \in \mathscr{D}$, as well as
  - treatment assignments, $D_i \in \mathscr{D}$.

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- Suppose $\mathscr{D} = \{0, 1\}$. Then under SUTVA, potential outcomes for an arbitrary draw from $\mathscr{P}$ are Y_{1i} and Y_{0i}.

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$$\rho_i = Y_{1i} - Y_{0i},$$

for which,

$$E[\rho_i] = E[Y_{1i} - Y_{0i}] = \rho, \tag{1}$$

the average treatment effect (ATE).

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- For our running example, we have $D_i = 1$ if college, $D_i = 0$ if not. Outcome of interest is income. ρ is the average income benefit of college.

Causal identification under the potential outcomes model

- Consider simple difference in mean college grad incomes vs mean no college incomes:

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

- Then what does this difference equal?

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- Could do similar wrt to ATC or effect heterogeneity (cf. CCI).

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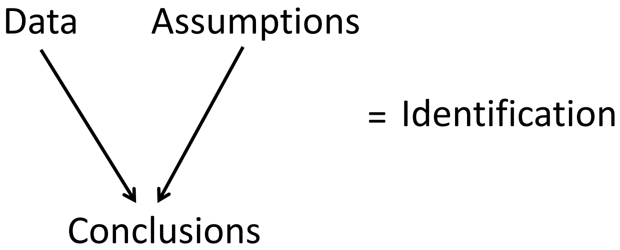

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As such,

$$\underbrace{E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1]}_{\text{ATT}} = E[Y_{1i} - Y_{0i}],$$

so the simple difference, (2), equals ρ.

Causal identification under the potential outcomes model

Identifying assumption 2 (conditionally independent/unconfounded/strongly ignorable assignment):

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i}) | X_i \text{ and } 0 < Pr[D_i = 1 | X_i = x] < 1 \text{ for all } x \in \mathscr{X} \quad (4)$$

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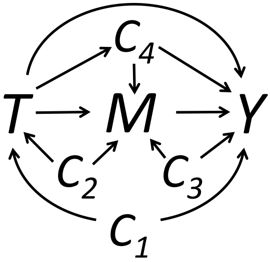
Marginalization over $\mathscr{X}$, the support of X_i, yields,

$$\int_{\mathscr{X}} \rho(x) dF(x) = \rho.$$



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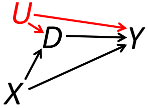

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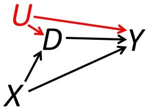

- Our target of inference is based on an hypothetical intervention:

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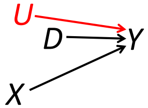

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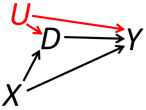

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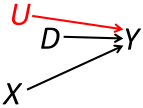

- Difference between $\mathrm{E}\left[Y|do(D_i = 1)\right] - \mathrm{E}\left[Y|do(D_i = 0)\right]$ and $\mathrm{E}\left[Y|D_i = 1\right] - \mathrm{E}\left[Y|D_i = 0\right]$ is "backdoor paths" from D to Y.

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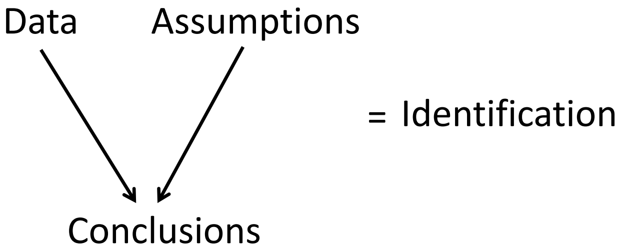

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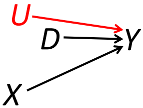

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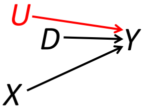

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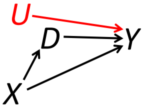

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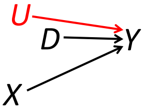

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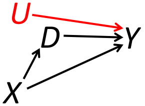
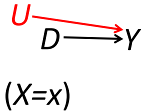


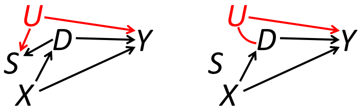
- Conditioning on X removes the other backdoor path:



- Marginalize over x to recover the intervention graph.

Causal graphs

- These are examples of "closing" backdoor paths.
- Other operations, e.g., "opening" backdoor paths by conditioning on "colliders":



Looking forward to the rest of the class

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  - These identifying assumptions rule out confounding by U_i.
  - If true, sufficient for identifying average treatment effect.
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- Time for questions.